Solving a convex quadratic maximization problem appearing in some distributionally robust problem

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ROADEF

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Context

$$\begin{aligned} \max_{x\in\mathbb{R}^n, y\in\mathbb{R}^m} \quad & d^\top y + c^\top x + \|x\|_2^2 \\ \text{s.t.} \quad & Ax + Ty = b, \\ & x \geq 0,\ y \geq 0 \end{aligned}$$

- → Appears as the subproblem in a Benders' decomposition for solving a Distributionally Robust Optimization (DRO) problem.
- → Optimality $\implies$ LB + UB
- → Problem is NP-Hard.
- → Generate some cuts and calculate the UB to prove the optimality is sufficient.

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Deterministic UC Problem

- $\mathcal{M}$: set of units
- T: time horizon
- $d \in \mathbb{R}^T$: demand vector
- x_i: commitment variables (binary).
- y_i: production variables (continuous).

$$
\begin{aligned}
\min_{x_i, y_i} \quad & \sum_{i \in \mathcal{M}} c_i^\top x_i + \sum_{i \in \mathcal{M}} b_i^\top y_i & \\
s.t. \quad & F_i x_i \geq f_i & \forall i \in \mathcal{M} \\
& H_i y_i \geq h_i & \forall i \in \mathcal{M} \\
& A_i x_i + B_i y_i \geq g_i & \forall i \in \mathcal{M} \\
& \sum_{i \in \mathcal{M}} y_i = d & \\
& x_i \in \{0,1\}^{m_i \times T}, \quad y_i \in \mathbb{R}_+^T &
\end{aligned}
$$

2-stage UC Problem under uncertainty

- Uncertainty on the demand.
- 2-stage assumption:
  - 1^{st} stage: commitment variables (binary)
  - 2^{nd} stage: production variables (continuous)

$$
\begin{aligned}
\min_{x_i, y_i} \quad & c^\top x + b^\top y \\
s.t. \quad & x \in X \\
& y_i \in Y_i(x_i) \qquad \forall i \in \mathcal{M} \\
& \sum_{i \in \mathcal{M}} y_i = \boxed{d}\,?
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- 2-stage assumption:
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$$\begin{aligned} \min_{x_i} \quad & c^\top x + \min_{\substack{y_i : y_i \in Y_i(x_i) \\ \sum_{i \in \mathcal{M}} y_i = d}} b^\top y \\ s.t. \quad & x \in X \end{aligned}$$

2-stage UC Problem under uncertainty

- Uncertainty on the demand.
- 2-stage assumption:
  - 1^{st} stage: commitment variables (binary)
  - 2^{nd} stage: production variables (continuous)

$$\min_{x_i} \quad c^\top x + Q(x,d)$$
$$s.t. \quad x \in X$$

$Q(x,d)$: recourse function representing the optimal cost of the second stage, considering which units are on or off (first stage) and the demand d

$$Q(x,d) = \min_{y} \quad b^\top y \qquad = \max_{\alpha,\beta,\gamma} \alpha^\top d + \beta^\top x + \gamma$$
$$y_i \in Y_i(x_i) \qquad (\alpha,\beta,\gamma) \in \Lambda$$
$$\sum_{i \in \mathcal{M}} y_i = d$$

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Distributionnally Robust Optimization

$$\begin{aligned} \min_x \quad & c^\top x + \max_{\mathbb{Q}\in\mathcal{P}} \mathbb{E}_{\xi\sim\mathbb{Q}}[Q(x,\xi)] \\ s.t. \quad & x_i \in X_i, \quad \forall i \in \mathcal{M} \end{aligned}$$

Link between RO and SP:

- If $\mathcal{P} = \{\mathbb{P}_0\}$, then the DRO problem is equal to the risk-neutral one.
- If $\mathcal{P}$ includes all distributions supported on $\mathcal{U}$, then the DRO problem is equivalent to the robust optimization (RO) problem with $\mathcal{U}$ as the uncertainty set.

$\Longrightarrow$ **How should we select $\mathcal{P}$?**

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DRO Unit Commitment problem

Idea: $\mathcal{P}$ has to include distributions "close" to the empirical one.
Wasserstein distance-based ambiguity sets

$$\mathcal{P} = \left\{ \mathbb{Q} \,|\, supp(\mathbb{Q}) \subset \mathbb{R}^T,\ W_2(\mathbb{Q}, \mathbb{P}_0) \leq \theta \right\}, \qquad \mathbb{P}_0 = \frac{1}{N} \sum_{i \in [N]} \delta_{\zeta_i}$$

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Reformulation DRO Unit Commitment problem[1]:

$$\begin{aligned} \min_x \quad & c^\top x + \max_{\substack{\mathbb{Q}:\ W_2(\mathbb{Q},\mathbb{P}_0) \leq \theta \quad [\lambda] \\ supp(\mathbb{Q}) \subset \mathbb{R}^T}} \mathbb{E}_{\mathbb{Q}}[Q(x,\xi)] \\ s.t. \quad & x \in X \end{aligned}$$

[1] Gao and Kleywegt 2016.

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Reformulation DRO Unit Commitment problem[1]:

$$\min_{x, \lambda \geq 0} \quad c^\top x + \lambda \theta^2 + \frac{1}{N} \sum_{i=1}^{N} \sup_{\xi \in \mathbb{R}^T} \left(Q(x, \xi) - \lambda \|\xi - \zeta_i\|_2^2 \right)$$

$$s.t. \quad x \in X$$

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Reformulation DRO Unit Commitment problem[1]:

$$\min_{x, \lambda \ge 0} \quad c^\top x + \lambda \theta^2 + \frac{1}{N} \sum_{i=1}^{N} \max_{\xi \in \mathbb{R}^T, k \in [K]} \left(\alpha_k^\top \xi + \beta_k^\top x + \gamma_k - \lambda \|\xi - \zeta_i\|^2 \right)$$

$$s.t. \quad x \in X$$

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Reformulation DRO Unit Commitment problem[1]:

$$\begin{aligned}
\min_{x, \lambda \geq 0, z \geq 0, w \geq 0} \quad & c^\top x + w\theta^2 + \frac{1}{N} \sum_{i=1}^{N} z_i \\
s.t. \quad & x \in X \\
& \|(2, w - \lambda)\|_2 \leq w + \lambda \\
& z_i \geq \left(\alpha_k^\top \zeta_i + \beta_k^\top x + \gamma_k + \frac{\lambda \|\alpha_k\|_2^2}{4} \right) \quad \forall k \in [K]
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$\Longrightarrow$ **Benders' algorithm**

[1] Gao and Kleywegt 2016.

Benders algorithm DRO

Master Problem:

$$\begin{aligned}
\min_{x,w,\lambda,z} \quad & c^\top x + w\theta^2 + \frac{1}{N}\sum_{i=1}^{N} z_i \\
s.t. \quad & x \in X \\
& \|(2, w-\lambda)\|_2 \le w + \lambda \\
& \boldsymbol{z_i \ge (\alpha_\ell^i)^\top \zeta^i + (\beta_\ell^i)^\top x + \gamma_\ell^i + \frac{\lambda \|\alpha_\ell^i\|_2^2}{4}, \quad \forall \ell \le k, \forall i \in [N]} \\
& \theta \in \mathbb{R}, \quad x_i \in \{0,1\}^{m_i \times T}
\end{aligned}$$

(x^*, λ^*)

$(\alpha^*, \beta^*, \gamma^*)$

Subproblem for each scenario *i*:

$$\begin{aligned}
\max_{\alpha,\beta,\gamma} \quad & \alpha^\top \zeta_i + \beta^\top x^* + \gamma + \frac{\lambda^* \|\alpha\|_2^2}{4} \\
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Benders algorithm DRO

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Convex maximization problem (NP-Hard)

Comparison risk-neutral/ DRO subproblems

Computational times on instance with 50 units and 1 random demand

$\implies$ dimension of the quadratic term: $T = 24$

Subproblem Risk-Neutral for each scenario i:

$$\max_{\alpha,\beta,\gamma} \quad \alpha^\top \zeta_i + \beta^\top x^* + \gamma$$
$$s.t. \quad (\alpha, \beta, \gamma) \in \Lambda$$

$$\downarrow$$

$$\boxed{\text{LP}}$$

$$\downarrow$$

$$\boxed{\approx 0.002s}$$

Subproblem DRO for each scenario i:

$$\max_{\alpha,\beta,\gamma} \quad \alpha^\top \zeta_i + \beta^\top x^* + \gamma + \frac{\lambda^* \|\alpha\|_2^2}{4}$$
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$$\boxed{\text{Convex maximization problem (NP-Hard)}}$$

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$$\boxed{\approx 0.5s}$$

- **Problem:** Directly solving it with Gurobi can be challenging.
- **Idea:** By using only "good" cuts (potentially suboptimal), we can often eliminate the current solution.

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Convex maximization problem: ("Hard" to solve)

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$\boxed{\text{Optimal value: } v^*, \qquad \text{Optimal solution: } (x^*, y^*)}$ ← Optimal cut

Linearization: ("Easy" to solve)

$$(\mathcal{P}_\ell(\bar{x})) \quad \begin{aligned} \max_{x\in\mathbb{R}^n, y\in\mathbb{R}^m} \quad & d^\top y + (c + 2\bar{x})^\top x - \|\bar{x}\|_2^2 \\ \text{s.t.} \quad & Ax + Ty = b, \\ & x \geq 0,\ y \geq 0 \end{aligned}$$

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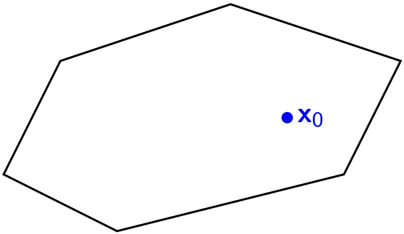
Frank-Wolfe Algorithm

$$v(x,y) = d^\top y + c^\top x + \|x\|_2^2$$



Choice of the step length α_k by solving:

$$\max_{\alpha \in [0,1]} v((x_k, y_k) + \alpha(p_k^x, p_k^y))$$

$\implies$ By convexity, $\alpha_k \in \{0,1\} \implies (x_{k+1}, y_{k+1}) = s^\ell(x_k)$.
$\implies$ The sequence $(v(x_k, y_k))_{k \geq 1}$ is strictly increasing.
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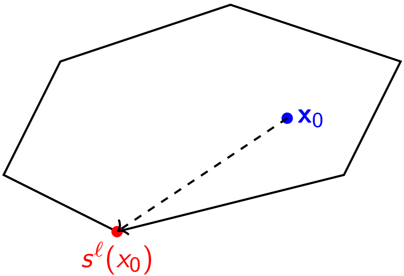

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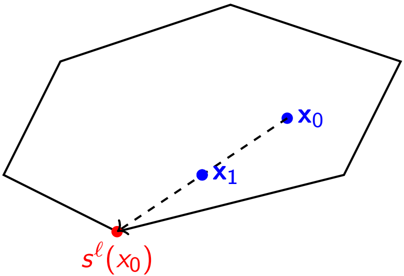

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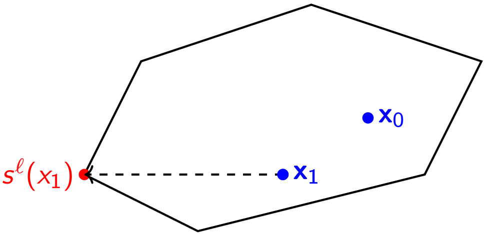

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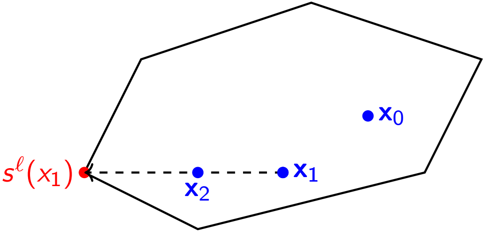

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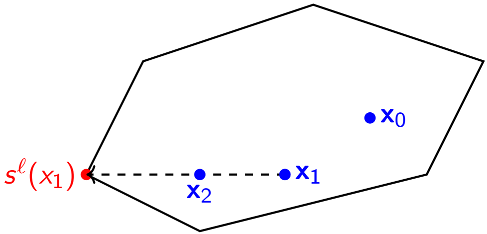

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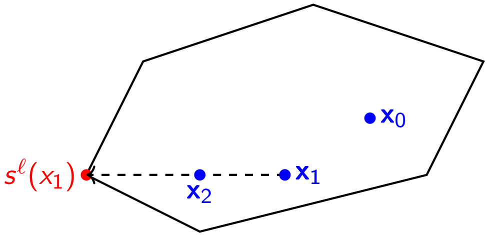
Frank-Wolfe Algorithm

$$v(x, y) = d^\top y + c^\top x + \|x\|_2^2$$



Choice of the step length α_k by solving:

$$\max_{\alpha \in [0,1]} v((x_k, y_k) + \alpha(p_k^x, p_k^y))$$

$\implies$ By convexity, $\alpha_k \in \{0, 1\} \implies (x_{k+1}, y_{k+1}) = s^\ell(x_k)$.

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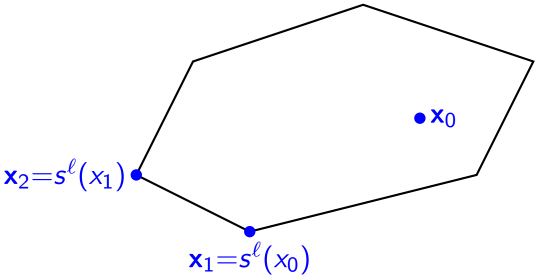

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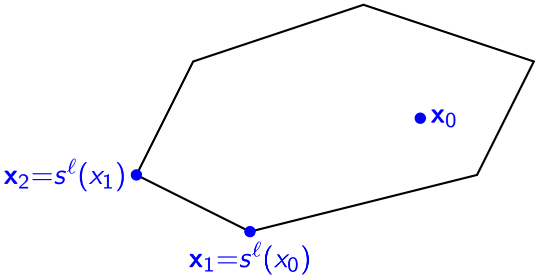

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Contents

1^{st} starting point strategy: Sampling

Attraction field of the face F:

$$P_x(F) = \{\bar{x} \quad \text{s.t.} \quad F \text{ is the set of optimal solutions of the linearized problem at } \bar{x}\}$$

Proposition

The set $P_x(F)$ is a polyhedron.

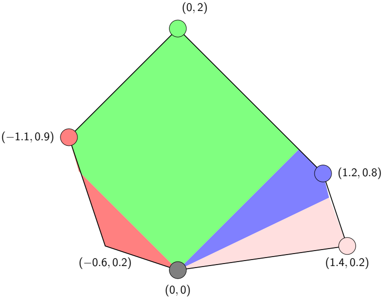

Figure: Maximizing $\|x\|_2^2$

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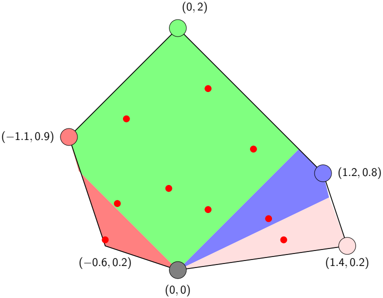
Proposition

The relative interior of $P_x(x^*)$ is non empty.

- If we randomly draw the starting point, FW algorithm can find the optimal solution with a non zero probability.



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The extreme point (x, y) is a local optimum of the quadratic convex maximization problem if and only if x belongs to the attraction field of (x, y).

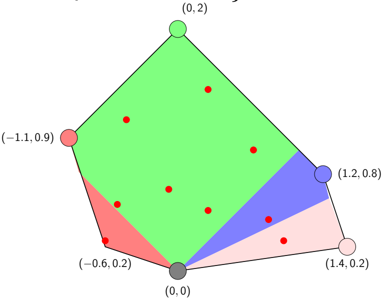

Figure: Maximizing $\|x\|_2^2$

$\Longrightarrow$ FW algorithm finds a local optimum.

2^{nd} starting point strategy: Discretize the space

Proposition

Let $\bar{x}$ and $v(x,y)$ defined by :

$$v(x,y) = d^\top y + c^\top x + \|x\|_2^2$$

the following inequality holds:

$$v^\ell(\bar{x}) \leq v(s^\ell(\bar{x})) \leq v^* \leq v^\ell(\bar{x}) + \|\bar{x} - x^*\|^2.$$

Corollary

Let $\varepsilon > 0$ and $X \subset \mathbb{R}^n$ such that:

$$\max_{x\in P} dist(x,X) \leq \varepsilon \qquad dist(x,X) = \min_{\bar{x}\in X} \|x - \bar{x}\|_2$$

Define $v^\ell(X) = \max_{x\in X} v^\ell(x)$. Then, the following inequalities hold:

$$v^\ell(X) \leq v^* \leq \varepsilon^2 + v^\ell(X)$$

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A MILP to solve the discretized problem

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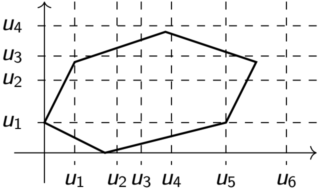

$$v^\ell(X) \leq v^* \leq v^\ell(X) + n^2 \max_{k \in N} \frac{|u_k - u_{k-1}|^2}{4}$$

$$X = \left\{ x \mid x_i = \sum_{k \in [N]} \delta^i_k u_k \ s.t. \left\{ \begin{array}{l} \delta^i_k \in \{0,1\} \\ \sum_{k \in [N]} \delta^i_k = 1 \end{array} \right. \right\}$$

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$\Longrightarrow$ Linearization through binary variables δ^i_k

$\Longrightarrow$ MILP

A MILP to solve the discretized problem

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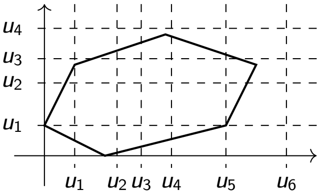

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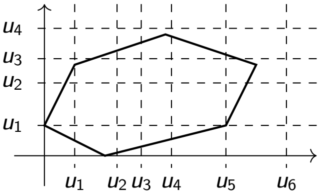

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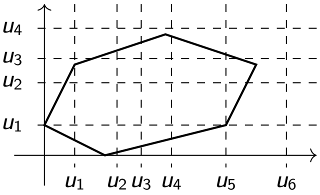

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Adapted Benders' Algorithm

First case

Draw a starting point

↓

Apply FW algorithm

↓

Local optimum

↓

New cut ?

Adapted Benders' Algorithm

First case

Draw a starting point

↓

Apply FW algorithm

↓

Local optimum

↓

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Second case

Solve disticretized problem (MILP)

↓

Apply FW algorithm

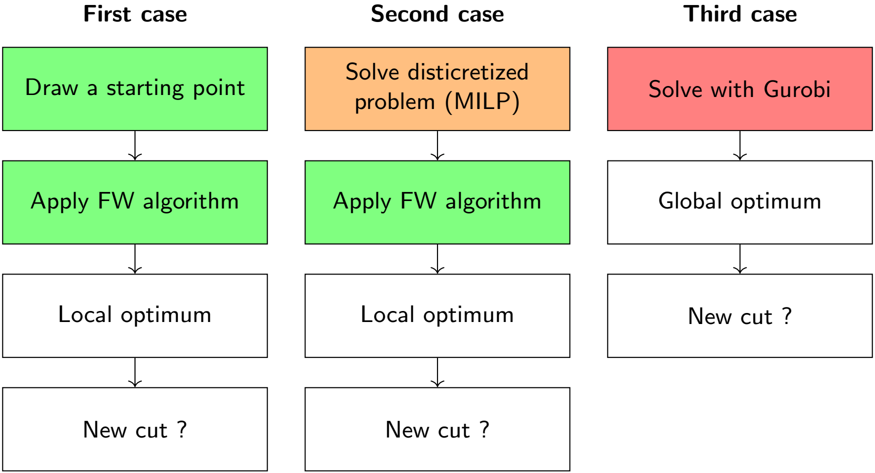
↓

Local optimum

↓

New cut ?

Adapted Benders' Algorithm



Adapted Benders' Algorithm

First case

Draw a starting point → Apply FW algorithm → Local optimum → New cut ?

No good UB

Second case

Solve disticretized problem (MILP) → Apply FW algorithm → Local optimum → New cut ?

No good UB

Third case

Solve with Gurobi → Global optimum → New cut ?

Good UB
but can be long

Contents

Contents

KKT reformulation

Structure of Oracle problem for the Unit commitment:

$$\begin{aligned} \max_{x\in\mathbb{R}^n, y\in\mathbb{R}^m_+} \quad & d^\top y + c^\top x + \|x\|_2^2 \\ \text{s.t.} \quad & x + T_i y_i = b_i, \qquad \forall i \\ & y \geq 0 \end{aligned}$$

- KKT conditions $\implies$ MILP.

→ **Advantage:** Provide optimal solution.

→ **Drawback:** Large number of binary variables.

→ **Worst than Gurobi solving directly the quadratic problem**

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PSD relaxation

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PSD relaxation first order:

$$\begin{aligned}\max_{x\in\mathbb{R}^n, y\in\mathbb{R}^m_+} \quad & d^\top y + c^\top x + Tr(X) \\ \text{s.t.} \quad & x + T_i y_i = b_i, \qquad \forall i \\ & y \geq 0,\ X = \begin{bmatrix} 1 & x^\top \\ x & xx^\top \end{bmatrix}\end{aligned}$$

PSD relaxation

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\max_{x\in\mathbb{R}^n, y\in\mathbb{R}^m_+} \quad & d^\top y + c^\top x + \|x\|_2^2 \\
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& y \geq 0
\end{aligned}
$$

PSD relaxation first order:

$$
\begin{aligned}
\max_{y\in\mathbb{R}^m_+} \quad & d^\top y + c^\top X_{1,.} + Tr(X) \\
\text{s.t.} \quad & X_{1,.} + T_i y_i = b_i, \qquad \forall i \\
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\end{aligned}
$$

→ **Advantage:** Tractable convex problem.

→ **Drawback:** Optimal value: $+\infty$

→ **Worst than Gurobi solving directly the quadratic problem**

PSD relaxation

$$\max_{x\in\mathbb{R}^n, y\in\mathbb{R}^m_+} \quad d^\top y + c^\top x + \|x\|_2^2$$
$$\text{s.t.} \quad x + T_i y_i = b_i, \qquad \forall i \tag{18a}$$
$$y \geq 0$$

PSD relaxation second order:

- **Idea:** Consider a large PSD matrix that include product variables between x and y up to order 2.
- Multiply constraint (18a) by each variable x_t, $y_{i,t}$

→ **Advantage:** Bounded convex problem and good relaxation.

→ **Drawback:** Untractable with solvers like Mosek. PSD size: $O((NT)^2)$

→ **Worst than Gurobi solving directly the quadratic problem**

PSD relaxation

$$\begin{aligned} \max_{x\in\mathbb{R}^n, y\in\mathbb{R}^m_+} \quad & d^\top y + c^\top x + \|x\|_2^2 \\ \text{s.t.} \quad & x + T_i y_i = b_i, \qquad \forall i \\ & y \geq 0 \end{aligned}$$

Smaller PSD relaxation second order:

- **Idea:** Use smaller PSD matrices that include product variables between x_t and y up to order 2.

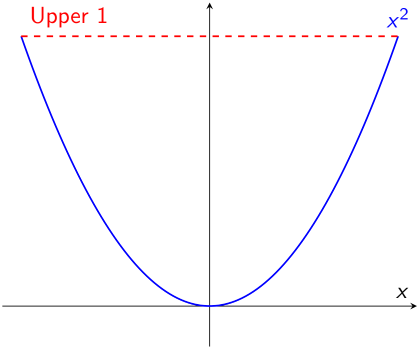
$\rightarrow$ **Advantage:** Bounded convex problem and good relaxation.

$\rightarrow$ **Drawback:** Untractable with solvers like Mosek. T PSD of size: $O(N^2)$

$\rightarrow$ **Worst than Gurobi solving directly the quadratic problem**

Contents

Piecewise upper approximation



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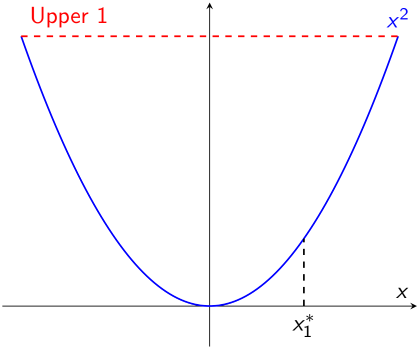

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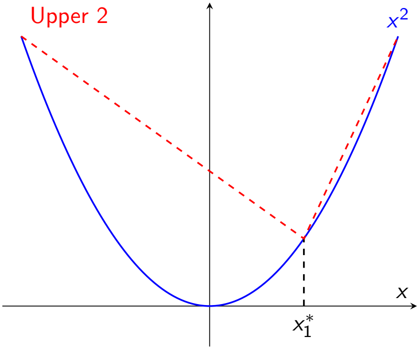

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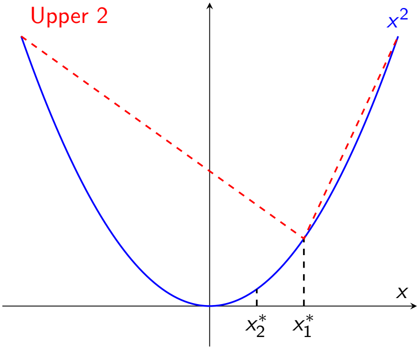

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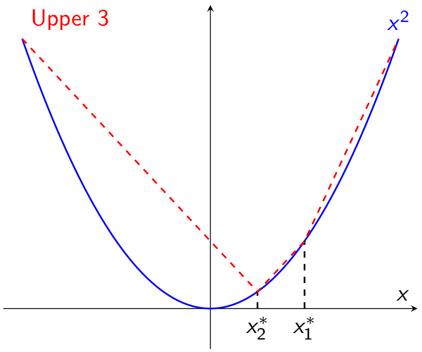

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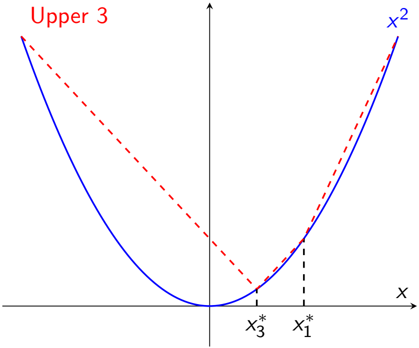

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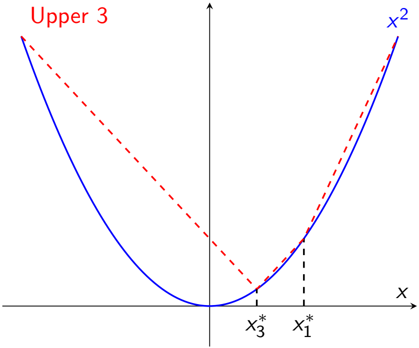
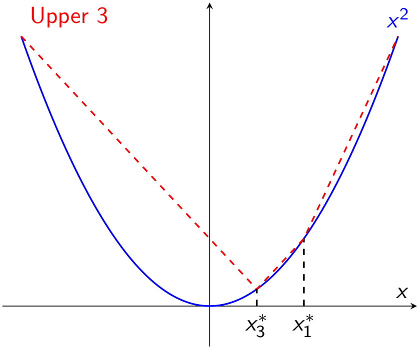


- MILPs.

Piecewise upper approximation



- MILPs.
- Number of binary variables increase at each iteration

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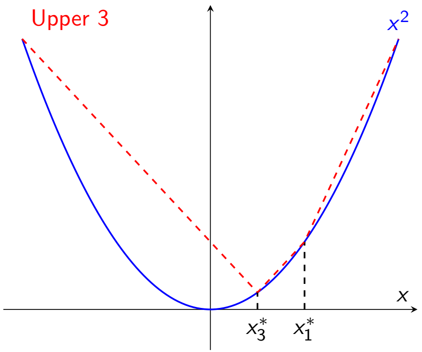
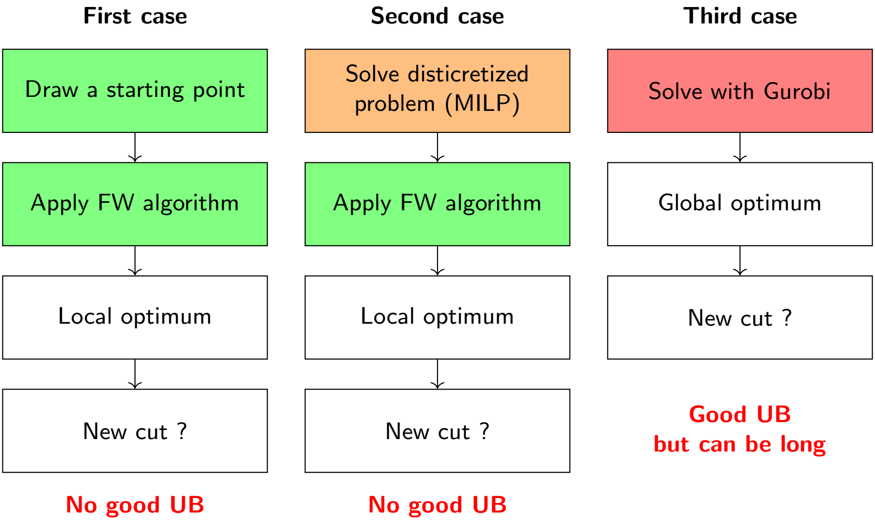


- MILPs.
- Number of binary variables increase at each iteration
- Convergence in few iterations.

Adapted Benders' Algorithm



Adapted Benders' Algorithm

First case

1. Draw a starting point
2. Apply FW algorithm
3. Local optimum
4. New cut ?

Second case

1. Solve disticretized problem (MILP)
2. Apply FW algorithm
3. Local optimum
4. New cut ?

Third case

1. Piecewise upper approximation (MILP)
2. Global optimum
3. New cut ?

Contents

Instances

- Source: SMS++/ EDF. Instances with generators (10, 20, 50) and only one random demand (dimension: $T = 24$)
- Source IEEE: 2 instances with network constraints:
  - 14 thermal units and 4 buses $\implies$ dimension of uncertainty: $4 \times T = 96$
  - 54 generators and 118 buses $\implies$ dimension of uncertainty: $118 \times T = 2832$

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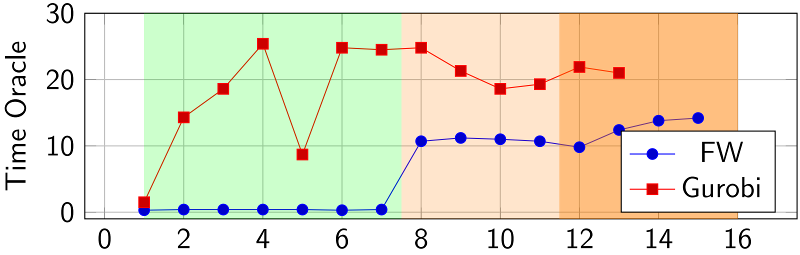
	SMS10	SMS20	SMS50	IEEE14	IEEE54
Units	10	20	50	14	54
ξ dimension	24	24	24	96	2832
`FW`	0.002	0.002	0.002	0.002	0.1
`MILP start`	0.03	0.05	0.4	0.05	15
`MILP UB`	0.1	0.07	0.5	0.05	15
`Gurobi`	0.2	0.5	0.75	0.1	15

Table: Time comparison (s)

Time saved at each iteration $\times$ Number of scenarios !

Convergence Analysis



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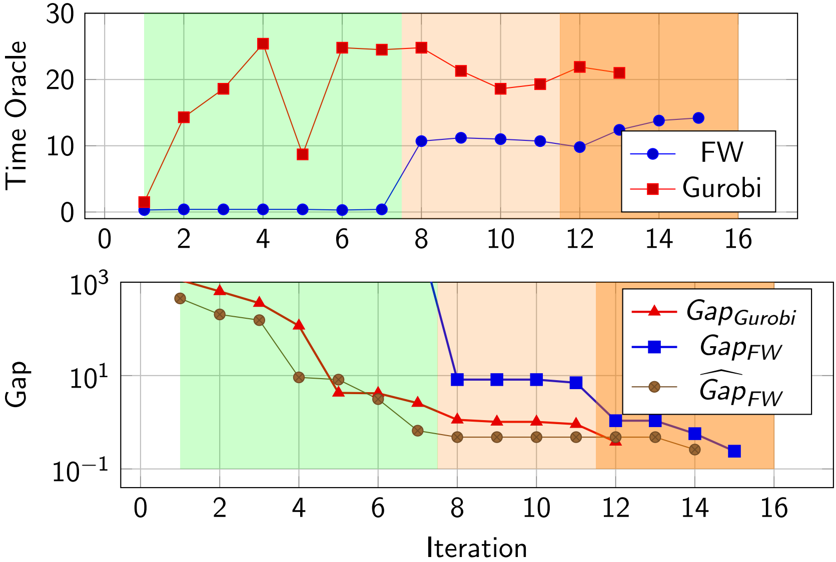

Conclusion

Summary

- Analysis of an NP-hard problem that arises as the oracle problem in a Benders' decomposition for solving a DRO problem.
- Show how the Frank-Wolfe algorithm can be efficient to progress in the Benders' algorithm by generating good cuts.
- Exploration of methods for calculating upper bounds.

Future works

- Addressing high-dimensional uncertainty (e.g., complex networks in Unit Commitment).
- Improving the formulation of the master problem.
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