Enhanced Benders' Decomposition for Stochastic Unit Commitment Using Interval Variables

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CERMICS
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EDF R&D

Outline

Contents

Description

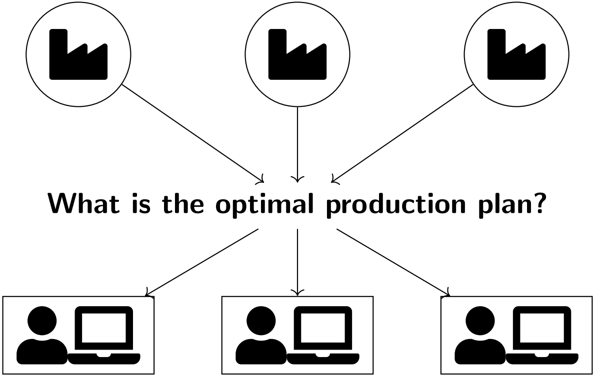

Figure: Unit Commitment

- $\mathcal{G}$: set of generators
- T: time horizon (T $=$ 24 for one day)
- $\xi \in \mathbb{R}^T$: vector of demand

Variables

- $\boldsymbol{p_{g,t}}$: power generation of generator g at time t.

$$\sum_{g \in \mathcal{G}} p_g = \xi$$

- $\boldsymbol{x_{g,t}} = (\boldsymbol{u_{g,t}}, \boldsymbol{v_{g,t}}, \boldsymbol{w_{g,t}}) \in \{0,1\}^3$
  - $\boldsymbol{u_{g,t}}$: equals 1 if generator g is on at time t.
  - $\boldsymbol{v_{g,t}}$: equals 1 if generator g starts up at time t.
  - $\boldsymbol{w_{g,t}}$: equals 1 if generator g shuts down at time t.

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\boldsymbol{u_{g,t}}$	0	0	1	1	1	0	0	1	1	0	1	1	1	0	0
$\boldsymbol{v_{g,t}}$	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0
$\boldsymbol{w_{g,t}}$	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0

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$\boldsymbol{v_{g,t}}$	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0
$\boldsymbol{w_{g,t}}$	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0

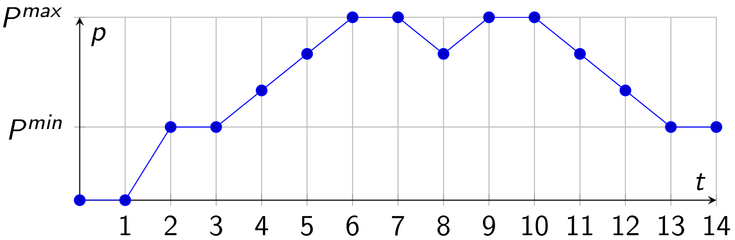
Technical Constraints

Constraints linking the variables x and p

$$W_g x_g \leq A_g p_g$$

Example:

- Capacity constraints: $P_g^{min} u_{g,t} \leq p_{g,t} \leq P_g^{max} u_{g,t}$



u_t	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1

Technical Constraints

Constraints linking the variables x and p

$$W_g x_g \leq A_g p_g \quad \forall g$$

Constraints on the commitment variables $x = (u, v, w)$

$$\begin{aligned} & u_{g,t} - u_{g,t-1} = v_{g,t} - w_{g,t} && \forall g, \forall t \\ & x_g \in X_g && \forall g \end{aligned}$$

Minimum uptime constraint in X_g:

$$\sum_{k \in [t - T_g^U + 1, t]} v_{g,k} \leq u_{g,t}$$

Technical Constraints

Constraints linking the variables x and p

$$W_g x_g \leq A_g p_g \quad \forall g$$

Constraints on the commitment variables $x = (u, v, w)$

$$u_{g,t} - u_{g,t-1} = v_{g,t} - w_{g,t} \qquad \forall g, \forall t$$
$$x_g \in X_g \qquad \forall g$$

Demand satisfaction:

$$\sum_{g \in \mathcal{G}} p_g = \xi$$

3-bin formulation

$$
\begin{aligned}
\min_{x,p} \quad & \sum_{g\in\mathcal{G},\, t\in[T]} C_{g,t}^{\top} x_{g,t} + \sum_{g\in\mathcal{G},\, t\in[T]} c_{g,t}\, p_{g,t} \\
\text{s.t.} \quad & \sum_{g\in\mathcal{G}} p_g = \xi \\
& W_g x_g \leq A_g p_g && \forall g \\
& u_{g,t} - u_{g,t-1} = v_{g,t} - w_{g,t} && \forall g, \forall t \\
& x_g \in X_g && \forall g \\
& x_{g,t} = (u_{g,t}, v_{g,t}, w_{g,t}) \\
& x_g \in \{0,1\}^{3T},\ p_g \in \mathbb{R}^T.
\end{aligned}
$$

3-bin formulation

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\min_{x,p} \quad & \sum_{g\in\mathcal{G},\, t\in[T]} C_{g,t}^{\top} x_{g,t} + \sum_{g\in\mathcal{G},\, t\in[T]} c_{g,t}\, p_{g,t} \\
\text{s.t.} \quad & \sum_{g\in\mathcal{G}} p_g = \xi \\
& W_g x_g \leq A_g p_g & \forall g \\
& x \in X,\ p_g \in \mathbb{R}^T.
\end{aligned}
$$

Contents

2-stage Stochastic Unit Commitment

$$\begin{aligned}\min_{x,p} \quad & \sum_{g,t} C_{g,t}^{\top} x_{g,t} + \sum_{g,t} c_{g,t}\, p_{g,t} \\ \text{s.t.} \quad & \sum_{g} p_g = \boxed{\xi}\,? \\ & W_g x_g \leq A_g p_g \qquad \forall g \\ & x \in X,\ p_g \in \mathbb{R}^T. \end{aligned}$$

- Uncertainty on the demand ξ
- **2-stage assumption:**
  - **First-stage:** Commitment decisions $x_{g,t}$ made before uncertainty is revealed
  - **Second-stage:** Generation decisions $p_{g,t}$ made after uncertainty is revealed
- Set of scenarios ξ_s with probability π_s

2-stage Stochastic Unit Commitment

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\min_{x,p} \quad & \sum_{g,t} C_{g,t}^{\top} x_{g,t} + \sum_{s \in \mathcal{S}} \pi_s \sum_{g,t} c_{g,t}\, p_{s,g,t} \\
\text{s.t.} \quad & \sum_{g} p_{s,g} = \xi_s \quad \forall s \\
& W_g x_g \leq A_g p_{s,g} \quad \forall g, \forall s \\
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$\Longrightarrow$ **Extensive formulation: 1 Mixed-Integer Linear Program**

Recourse function: Primal formulation

$Q(x,\xi)$: recourse value representing the optimal cost of the second stage

$$Q(x,\xi) = \min_{p} \sum_{g,\,t} c_{g,t}\, p_{g,t}$$
$$\text{s.t.} \sum_{g} p_g = \xi$$
$$W_g x_g \leq A_g p_g \quad \forall g$$

Recourse function: Primal formulation

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& W_g x_g \le A_g p_{s,g} \quad \forall g, \forall s \\
& x \in X
\end{aligned}
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\end{aligned}
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$$
\begin{aligned}
\min_{x} \; & \sum_{g,t} C_{g,t}^{\top} x_{g,t} \\
& + \sum_{s\in\mathcal{S}} \pi_s \min_{\substack{p_s : \sum_g p_{s,g} = \xi_s \\ W_g x_g \le A_g p_{s,g}}} \sum_{g,t} c_{g,t}\, p_{g,t,s} \\
\text{s.t. } & x \in X
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$$\min_{x} \sum_{g,t} C_{g,t}^{\top} x_{g,t} + \sum_{s \in \mathcal{S}} \pi_s Q(x, \xi_s)$$
$$\text{s.t. } x \in X$$

Recourse Function: Dual formulation

$Q(x,\xi)$: recourse value representing the optimal cost of the second stage

Primal formulation:

$$\min_{p} \sum_{g,t} c_{g,t}\, p_{g,t}$$

$$\text{s.t.} \sum_{g} p_g = \xi \qquad [\nu]$$

$$W_g x_g \le A_g p_g \;\forall g \;[\mu_g]$$

Dual formulation:

$$\max_{\nu,\mu} \nu^\top \xi + \sum_{g} \mu_g^\top W_g x_g$$

$$\text{s.t. } A_g^\top \mu_g + \nu = c_g \quad \forall g$$

$$\mu_g \ge 0 \quad \forall g$$

Recourse Function: Dual formulation

$\boldsymbol{Q(x,\xi)}$: **recourse value representing the optimal cost of the second stage**

Primal formulation:

$$\min_{p} \sum_{g,t} c_{g,t}\, p_{g,t}$$
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Independent of x and ξ !

Recourse Function: Dual formulation

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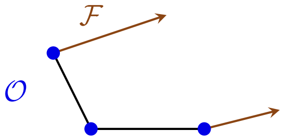

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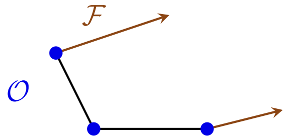

$$Q(x,\xi) = \begin{cases} +\infty & \text{if } \exists f \in \mathcal{F}, (\nu^f)^\top \xi + \sum_g (\mu_g^f)^\top W_g x_g > 0 \\ \max_{o \in \mathcal{O}} \left((\nu^o)^\top \xi + \sum_g (\mu_g^o)^\top W_g x_g \right) & \text{otherwise.} \end{cases}$$

3-bin Benders' Decomposition

$$\min_x \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s Q(x,\xi_s)$$

s.t. $x \in X$

$$Q(x,\xi) = \begin{cases} +\infty & \text{if } \exists f \in \mathcal{F}, (\nu^f)^\top \xi + \sum_g (\mu_g^f)^\top W_g x_g > 0 \\ \max_{o \in \mathcal{O}} \left((\nu^o)^\top \xi + \sum_g (\mu_g^o)^\top W_g x_g \right) & \text{otherwise.} \end{cases}$$

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$$\min_x \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s z_s$$
$$\text{s.t. } x \in X$$
$$z_s \geq Q(x,\xi_s) \qquad \forall s$$

3-bin Benders' Decomposition

$$\min_x \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s Q(x,\xi_s)$$
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$$\min_x \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s z_s$$
$$\text{s.t. } x \in X$$
$$0 \geq (\nu^f)^\top \xi_s + \sum_g (\mu_g^f)^\top W_g x_g \quad \forall s, \forall f \in \mathcal{F}$$

3-bin Benders' Decomposition

$$\min_x \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s Q(x, \xi_s)$$

s.t. $x \in X$

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s.t. $x \in X$

$$0 \geq (\nu^f)^\top \xi_s + \sum_g (\mu_g^f)^\top W_g x_g \quad \forall s, \forall f \in \mathcal{F}$$

$$z_s \geq (\nu^o)^\top \xi_s + \sum_g (\mu_g^o)^\top W_g x_g \quad \forall s, \forall o \in \mathcal{O}$$

$\Longrightarrow$ **Iterative generation of these feasibility and optimality cuts until convergence!**

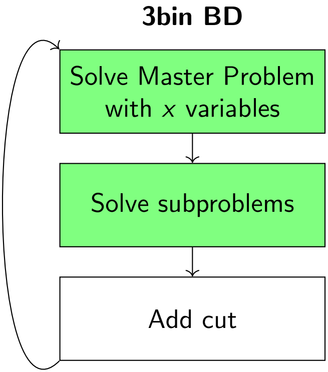
3-bin Benders' Decomposition

$$\min_x \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s z_s$$
$$\text{s.t. } x \in X$$
$$0 \geq (\nu^f)^\top \xi_s + \sum_g (\mu_g^f)^\top W_g x_g \qquad \forall s, \forall f \in \mathcal{F}$$
$$z_s \geq (\nu^o)^\top \xi_s + \sum_g (\mu_g^o)^\top W_g x_g \qquad \forall s, \forall o \in \mathcal{O}$$

Subproblem to generate cuts for scenario s:

$$\min_p \sum_{g,t} c_{g,t}\, p_{g,t}$$
$$\text{s.t. } \sum_g p_g = \xi \qquad [\nu]$$
$$W_g x_g \leq A_g p_g \; \forall g \; [\mu_g]$$

Algorithm


3bin BD
Solve Master Problem with x variables
Solve subproblems
Add cut

Analysis of the weakness of Benders' Cuts

$$z_s \geq \underbrace{\boxed{(\nu^o)^\top \xi_s}}_{\text{Intercept}} + \sum_g \underbrace{\boxed{(\mu_g^o)^\top W_g x_g}}_{\text{Contribution of unit } g}$$

$$\begin{aligned} \max_{\nu,\mu}\ & \nu^\top \xi_s + \sum_g \mu_g^\top W_g x_g \\ \text{s.t. } & A_g^\top \mu_g + \nu = c_g \quad \forall g \\ & \mu_g \geq 0 \quad \forall g \end{aligned} \quad \longrightarrow \quad \begin{aligned} \max_{\mu_g}\ & \mu_g^\top W_g x_g \\ \text{s.t. } & A_g^\top \mu_g = c_g - \nu^o \\ & \mu_g \geq 0 \end{aligned}$$

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- The optimal solution μ_g^o depends on x_g.
- Even if it is optimal for a given x_g, it may be very suboptimal for another x_g'.
- Let us consider an example...

Analysis of the weakness of Benders' Cuts

$$\begin{aligned} \max_{\mu_g} \ & \mu_g^\top W_g x_g \\ \text{s.t. } & A_g^\top \mu_g = c_g - \nu^o \\ & \mu_g \geq 0 \end{aligned} \quad \longleftrightarrow \quad \begin{aligned} \min_{p_g} \ & (c_g - \nu^o)^\top p_g \\ \text{s.t. } & W_g x_g \leq A_g p_g \end{aligned}$$

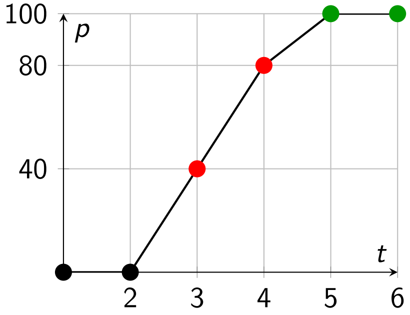
Consider an example where $c_g - \nu^o = -10$ and a unit g with the following constraints:

$$\begin{aligned} p_{g,t} &\leq 100 x_{g,t} & [\mu_{g,t}^{max}] \\ p_{g,t} &\geq 40 x_{g,t} & [\mu_{g,t}^{min}] \\ p_{g,t} - p_{g,t-1} &\leq 40 x_{g,t} & [\mu_{g,t}^{up}] \\ p_{g,t-1} - p_{g,t} &\leq 40 x_{g,t} & [\mu_{g,t}^{down}] \end{aligned}$$

Analysis of the weakness of Benders' Cuts

$$\max_{\mu_g} \mu_g^\top W_g x_g \quad \text{s.t. } A_g^\top \mu_g = c_g - \nu^o, \quad \mu_g \geq 0 \quad \longleftrightarrow \quad \min_p \sum_{t\in[T]} -10p_t \quad \text{s.t. } 40x_t \leq p_t \leq 100x_t, \quad -40x_t \leq p_t - p_{t-1} \leq 40x_t$$

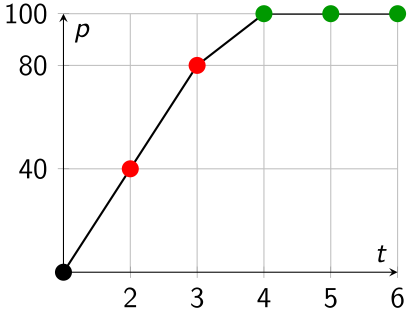
Iteration 1							
		1	2	3	4	5	6
	x_t^1	0	0	1	1	1	1
μ^1	μ_t^{max}	10	30	0	0	10	10
	μ_t^{min}	0	0	0	0	0	0
	μ_t^{up}	0	0	20	10	0	0
	μ_t^{down}	0	0	0	0	0	0
$(\mu^1)^\top W x^1 = -3200$							



Analysis of the weakness of Benders' Cuts

$$\begin{aligned}\max_{\mu_g}\ & \mu_g^\top W_g x_g \\ \text{s.t. } & A_g^\top \mu_g = c_g - \nu^o \\ & \mu_g \geq 0\end{aligned} \quad \longleftrightarrow \quad \begin{aligned}\min_p\ & \sum_{t\in[T]} -10p_t \\ \text{s.t. } & 40x_t \leq p_t \leq 100x_t \\ & -40x_t \leq p_t - p_{t-1} \leq 40x_t\end{aligned}$$

Iteration 2							
		1	2	3	4	5	6
	x_t^2	0	1	1	1	1	1
μ^1	μ_t^{max}	30	0	0	10	10	10
	μ_t^{min}	0	0	0	0	0	0
	μ_t^{up}	0	20	10	0	0	0
	μ_t^{down}	0	0	0	0	0	0
$(\mu^2)^\top Wx^2 = -4200$							



Analysis of the weakness of Benders' Cuts

$$\max_{\mu_g} \mu_g^\top W_g x_g \quad \text{s.t. } A_g^\top \mu_g = c_g - \nu^o, \quad \mu_g \geq 0 \quad \longleftrightarrow \quad \min_p \sum_{t\in[T]} -10p_t \quad \text{s.t. } 40x_t \leq p_t \leq 100x_t, \quad -40x_t \leq p_t - p_{t-1} \leq 40x_t$$

Iteration 1								Iteration 2							
		1	2	3	4	5	6			1	2	3	4	5	6
	x_t^1	0	0	1	1	1	1		x_t^2	0	1	1	1	1	1
μ^1	μ_t^{max}	10	30	0	0	10	10	μ^2	μ_t^{max}	30	0	0	10	10	10
	μ_t^{min}	0	0	0	0	0	0		μ_t^{min}	0	0	0	0	0	0
	μ_t^{up}	0	0	20	10	0	0		μ_t^{up}	0	20	10	0	0	0
	μ_t^{down}	0	0	0	0	0	0		μ_t^{down}	0	0	0	0	0	0
$(\mu^1)^\top W x^1 = -3200$								$(\mu^2)^\top W x^2 = -4200$							

Analysis of the weakness of Benders' Cuts

$$\max_{\mu_g} \mu_g^\top W_g x_g \quad \text{s.t. } A_g^\top \mu_g = c_g - \nu^o, \quad \mu_g \geq 0 \quad \longleftrightarrow \quad \min_p \sum_{t\in[T]} -10p_t \quad \text{s.t. } 40x_t \leq p_t \leq 100x_t, \quad -40x_t \leq p_t - p_{t-1} \leq 40x_t$$

Iteration 1								Iteration 2							
		1	2	3	4	5	6			1	2	3	4	5	6
	x_t^1	0	0	1	1	1	1		x_t^2	0	1	1	1	1	1
μ^1	μ_t^{max}	10	30	0	0	10	10	μ^2	μ_t^{max}	30	0	0	10	10	10
	μ_t^{min}	0	0	0	0	0	0		μ_t^{min}	0	0	0	0	0	0
	μ_t^{up}	0	0	20	10	0	0		μ_t^{up}	0	20	10	0	0	0
	μ_t^{down}	0	0	20	10	0	0		μ_t^{down}	0	0	0	0	0	0
$(\mu^1)^\top W x^1 = -3200$								$(\mu^2)^\top W x^2 = -4200$							
								$(\mu^1)^\top W x^2 = -6200$							

Analysis of the weakness of Benders' Cuts

$$\max_{\mu_g} \mu_g^\top W_g x_g \quad \text{s.t. } A_g^\top \mu_g = c_g - \nu^o, \quad \mu_g \geq 0$$

$\longleftrightarrow$

$$\min_p \sum_{t \in [T]} -10 p_t \quad \text{s.t. } 40x_t \leq p_t \leq 100x_t, \quad -40x_t \leq p_t - p_{t-1} \leq 40x_t$$

Iteration 1								Iteration 2							
		1	2	3	4	5	6			1	2	3	4	5	6
	x_t^1	0	0	1	1	1	1		x_t^2	0	1	1	1	1	1
μ^1	μ_t^{max}	10	30	0	0	10	10	μ^2	μ_t^{max}	30	0	0	10	10	10
	μ_t^{min}	0	0	0	0	0	0		μ_t^{min}	0	0	0	0	0	0
	μ_t^{up}	0	0	20	10	0	0		μ_t^{up}	0	20	10	0	0	0
	μ_t^{down}	0	0	20	10	0	0		μ_t^{down}	0	0	0	0	0	0
$(\mu^1)^\top W x^1 = -3200$								$(\mu^2)^\top W x^2 = -4200$							
								$(\mu^1)^\top W x^2 = -6200$							

Analysis of the weakness of Benders' Cuts

Iteration 1								**Iteration 2**							
		1	2	3	4	5	6			1	2	3	4	5	6
	x_t^1	0	0	1	1	1	1		x_t^2	0	1	1	1	1	1
μ^1	μ_t^{max}	10	30	0	0	10	10	μ^2	μ_t^{max}	30	0	0	10	10	10
	μ_t^{min}	0	0	0	0	0	0		μ_t^{min}	0	0	0	0	0	0
	μ_t^{up}	0	0	20	10	0	0		μ_t^{up}	0	20	10	0	0	0
	μ_t^{down}	0	0	0	0	0	0		μ_t^{down}	0	0	0	0	0	0
$(\mu^1)^\top W x^1 = -3200$								$(\mu^2)^\top W x^2 = -4200$							
								$(\mu^1)^\top W x^2 = -6200$							

In this example, we don't use the knowledge that after a unit starts up, it cannot reach its maximum output for two time periods.

Analysis of the weakness of Benders' Cuts

Another weakness is that these cuts suffer from **symmetries**.

Analysis of the weakness of Benders' Cuts

Another weakness is that these cuts suffer from **symmetries**.

Unit 1								**Unit 2**								
		1	2	3	4	5	6			1	2	3	4	5	6	
	x_t^1	0	0	1	1	1	1		x_t^2	0	1	1	1	1	1	
μ^1	μ_t^{max}	10	30	0	0	10	10	μ^2	μ_t^{max}	30	0	0	10	10	10	
	μ_t^{min}	0	0	0	0	0	0		μ_t^{min}	0	0	0	0	0	0	
	μ_t^{up}	0	0	20	10	0	0		μ_t^{up}	0	20	10	0	0	0	
	μ_t^{down}	0	0	0	0	0	0		μ_t^{down}	0	0	0	0	0	0	
$(\mu^1)^\top W x^1 = -3200$								$(\mu^2)^\top W x^2 = -4200$								-7400

Analysis of the weakness of Benders' Cuts

Another weakness is that these cuts suffer from **symmetries**.

		Unit 1								Unit 2						
		1	2	3	4	5	6			1	2	3	4	5	6	
	x_t^1	0	0	1	1	1	1		x_t^2	0	1	1	1	1	1	
μ^1	μ_t^{max}	10	30	0	0	10	10	μ^2	μ_t^{max}	30	0	0	10	10	10	
	μ_t^{min}	0	0	0	0	0	0		μ_t^{min}	0	0	0	0	0	0	
	μ_t^{up}	0	0	20	10	0	0		μ_t^{up}	0	20	10	0	0	0	
	μ_t^{down}	0	0	0	0	0	0		μ_t^{down}	0	0	0	0	0	0	
$(\mu^1)^\top W x^1 = -3200$								$(\mu^2)^\top W x^2 = -4200$								-7400
$(\mu^1)^\top W x^2 = -6200$								$(\mu^2)^\top W x^1 = -2400$								-8600

Contents

Addition of Interval First-Stage Variables

3-bin formulation

$\mathbf{x_{g,t} = (u_{g,t}, v_{g,t}, w_{g,t})} \in \{0,1\}^3$

- $u_{g,t}$: Binary variable equal to 1 if generator g is on at time t.
- $v_{g,t}$: Binary variable equal to 1 if generator g starts up at time t.
- $w_{g,t}$: Binary variable equal to 1 if generator g shuts down at time t.

Extended formulation

$\gamma_{g,a,b}$: equals 1 if generator g starts up at time a and shuts down at time b

$$u_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b}$$

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$u_{g,t}$	0	0	1	1	1	0	0	1	1	0	1	1	1	0	0

$\gamma_{g,2,5} = 1 \quad \gamma_{g,7,9} = 1 \quad \gamma_{g,9,13} = 1$

Addition of Interval First-Stage Variables

3-bin formulation

$\boldsymbol{x_{g,t} = (u_{g,t}, v_{g,t}, w_{g,t})} \in \{0,1\}^3$

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Extended formulation

$\boldsymbol{\gamma_{g,a,b}}$: equals 1 if generator g starts up at time a and shuts down at time b

$$\boldsymbol{u_{g,t}} = \sum_{[a,b]\ni t} \boldsymbol{\gamma_{g,a,b}}$$

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\boldsymbol{u_{g,t}}$	0	0	1	1	1	0	0	1	1	0	1	1	1	0	0

$\boldsymbol{\gamma_{g,2,5}} = 1 \quad \boldsymbol{\gamma_{g,7,9}} = 1 \quad \boldsymbol{\gamma_{g,9,13}} = 1$

Another formulation of the recourse problem

3-bin formulation

$$Q(\boldsymbol{x}, \xi) =$$

$$\min_{p} \sum_{g,t} c_{g,t}\, p_{g,t}$$

$$\text{s.t.} \sum_{g} p_g = \xi \qquad [\nu]$$

$$W_g \boldsymbol{x_g} \leq A_g p_g \; \forall g \; [\mu_g]$$

Extended formulation

$$\mathcal{Q}(\gamma, \xi) =$$

$$\min_{p} \sum_{g,[a,b],t} \gamma_{\boldsymbol{g,a,b}} c_{g,t}\, p_{g,a,b,t}$$

$$\text{s.t.} \sum_{g,[a,b]} \gamma_{\boldsymbol{g,a,b}} p_{g,a,b} = \xi \quad [\nu]$$

$$W_g x_{g,a,b} \leq A_g p_{g,a,b} \; \forall g \; \forall [a,b]$$

t	0	1	2	3	4	5	6	7	8	9	10
$x_{g,2,5,t}$	0	0	1	1	1	0	0	0	0	0	0

- $p_{g,a,b,t}$ represents the production of generator g at time t if it starts up at time a and shuts down at time b.
- $p_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b} p_{g,a,b,t}$

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$$Q(\boldsymbol{x}, \xi) =$$

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$$\mathcal{Q}(\gamma, \xi) =$$

$$\min_{p} \sum_{g,\,[a,b],t} \gamma_{\boldsymbol{g,a,b}} c_{g,t}\, p_{g,a,b,t}$$

$$\text{s.t.} \sum_{g,[a,b]} \gamma_{\boldsymbol{g,a,b}} p_{g,a,b} = \xi \quad [\nu]$$

$$W_g x_{g,a,b} \leq A_g p_{g,a,b} \ \forall g \ \forall [a, b]$$

t	0	1	2	3	4	5	6	7	8	9	10
$\boldsymbol{x_{g,2,5,t}}$	0	0	1	1	1	0	0	0	0	0	0

- $p_{g,a,b,t}$ represents the production of generator g at time t if it starts up at time a and shuts down at time b.
- $p_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b} p_{g,a,b,t}$

Another formulation of the recourse function

Extended formulation

$$\mathcal{Q}(\gamma, \xi) = \min_{p} \sum_{g,[a,b],t} \gamma_{\boldsymbol{g,a,b}} c_{g,t}\, p_{g,a,b,t}$$

$$\text{s.t.} \sum_{g,[a,b]} \gamma_{\boldsymbol{g,a,b}} p_{g,a,b} = \xi \qquad [\nu]$$

$$W_g x_{g,a,b} \leq A_g\, p_{g,a,b} \qquad \forall g\, \forall [a,b]$$

Another formulation of the recourse function

Extended formulation

$$\mathcal{Q}(\gamma, \xi) =$$

$$\max_{\nu} \min_{p} \sum_{g,\,[a,b],t} \gamma_{\boldsymbol{g,a,b}} c_{g,t}\, p_{g,a,b,t} + \nu^\top \left(\xi - \sum_{g,[a,b]} \gamma_{\boldsymbol{g,a,b}} p_{g,a,b} \right)$$

$$\text{s.t. } W_g x_{g,a,b} \leq A_g\, p_{g,a,b} \quad \forall g\, \forall [a,b]$$

Another formulation of the recourse function

Extended formulation

$$\mathcal{Q}(\gamma, \xi) = \max_{\nu} \nu^\top \xi + \min_{p} \sum_{g,\,[a,b],t} \gamma_{\boldsymbol{g,a,b}}(c_{g,t} - \nu_t)\, p_{g,a,b,t}$$

$$\text{s.t. } W_g x_{g,a,b} \leq A_g p_{g,a,b} \quad \forall g\, \forall [a, b]$$

Another formulation of the recourse function

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$$\mathcal{Q}(\gamma, \xi) = \max_{\nu} \nu^\top \xi + \sum_{g,\,[a,b]} \min_{p_{g,a,b}} \sum_t \gamma_{\boldsymbol{g,a,b}}(c_{g,t} - \nu_t)\, p_{g,a,b,t}$$

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$$\mathcal{Q}(\gamma,\xi) = \max_{\nu} \nu^\top \xi + \sum_{g,[a,b]} \gamma_{\boldsymbol{g,a,b}} \min_{p_{g,a,b}} \sum_t (c_{g,t} - \nu_t)\, p_{g,a,b,t}$$

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$$\mathcal{Q}(\gamma,\xi) = \max_{\nu} \nu^\top \xi + \sum_{g,[a,b]} \gamma_{\boldsymbol{g,a,b}} \min_{p_{g,a,b}} \sum_t (c_{g,t} - \nu_t)\, p_{g,a,b,t}$$

$$\text{s.t. } W_g x_{g,a,b} \leq A_g p_{g,a,b} \quad \forall g\, \forall [a,b]$$

$$\hat{Q}_{g,a,b}(y) = \min_{p \in \mathbb{R}^T} \quad y^\top p$$

$$W_g x_{g,a,b} \leq A_g p.$$

Another formulation of the recourse function

Extended formulation

$$\mathcal{Q}(\gamma, \xi) = \max_{\nu} \nu^\top \xi + \sum_{g,\,[a,b]} \boldsymbol{\gamma_{g,a,b}} \hat{Q}_{g,a,b}(c_g - \nu)$$

$$\hat{Q}_{g,a,b}(y) = \min_{p \in \mathbb{R}^T} \quad y^\top p$$
$$W_g x_{g,a,b} \leq A_g p.$$

A new formulation of the recourse function

Proposition

Under some weak assumptions, if x and γ represent the "same" first stage decisions, then:

$$Q(x,\xi) = \begin{cases} +\infty & \text{if } \exists f \in \mathcal{F}, (\nu^f)^\top \xi + \sum_g (\mu_g^f)^\top W_g x_g > 0 \\ \max_{o\in\mathcal{O}} \left((\nu^o)^\top \xi + \sum_g (\mu_g^o)^\top W_g x_g \right) & \text{otherwise.} \end{cases}$$

$$= \mathcal{Q}(\gamma,\xi) = \max_\nu \nu^\top \xi + \sum_{g,[a,b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(c_g - \nu)$$

$$= \begin{cases} +\infty & \text{if } \exists f \in \mathcal{F}, (\nu^f)^\top \xi + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(-\nu^f) > 0 \\ \max_{o\in\mathcal{O}} \left((\nu^o)^\top \xi + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(c_g - \nu^o) \right) & \text{otherwise.} \end{cases}$$

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Extended Benders' Decomposition (new)

$$
\begin{aligned}
&\min_{x,\gamma} \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_{s} \pi_s z_s \\
&\text{s.t. } x \in X, \quad u_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b} \quad \forall g, \forall t \\
&\qquad z_s \geq Q(x, \xi_s) \qquad \forall s \\
&\qquad 0 \geq (\nu^f)^\top \xi_s + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(-\nu^f) \qquad \forall s, \forall f \in \mathcal{F} \\
&\qquad z_s \geq (\nu^o)^\top \xi_s + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(c_g - \nu^o) \quad \forall s, \forall o \in \mathcal{O}
\end{aligned}
$$

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$$z_s \geq (\nu^o)^\top \xi_s + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(c_g - \nu^o) \quad \forall s, \forall o \in \mathcal{O}$$

- The cut coefficients $\hat{Q}_{g,a,b}$ contain **a lot of information** such that a generator cannot reach its maximum output before a certain number of time periods after starting up.

Extended Benders' Decomposition (new)

$$
\begin{aligned}
\min_{x,\gamma} \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_{s} \pi_s z_s \\
\text{s.t. } x \in X, \quad u_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b} \quad \forall g, \forall t \\
0 \geq (\nu^f)^\top \xi_s + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(-\nu^f) \qquad \forall s, \forall f \in \mathcal{F} \\
z_s \geq (\nu^o)^\top \xi_s + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(c_g - \nu^o) \quad \forall s, \forall o \in \mathcal{O}
\end{aligned}
$$

- It **does not suffer from symmetries**, *i.e.*, if two generators are identical, they will have the same cut coefficients.

Extended Benders' Decomposition (new)

$$\min_{x,\gamma} \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s z_s$$

$$\text{s.t. } x \in X, \quad u_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b} \quad \forall g, \forall t$$

$$0 \geq (\nu^f)^\top \xi_s + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(-\nu^f) \qquad \forall s, \forall f \in \mathcal{F}$$

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$$\min_p \sum_{g,t} c_{g,t}\, p_{g,t}$$

$$\text{s.t.} \sum_g p_g = \xi \qquad [\nu]$$

$$W_g \boldsymbol{x_g} \leq A_g p_g \, \forall g \, [\mu_g]$$

$$\min_p \sum_{g,[a,b],t} \boldsymbol{\gamma_{g,a,b}} c_{g,t}\, p_{g,a,b,t}$$

$$\text{s.t.} \sum_{g,[a,b]} \boldsymbol{\gamma_{g,a,b}} p_{g,a,b} = \xi \quad [\nu]$$

$$W_g x_{g,a,b} \leq A_g p_{g,a,b} \, \forall g \, \forall [a,b]$$

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$$\min_{x,\gamma} \sum_{g,t} C_{g,t}^{\top} x_{g,t} + \sum_{s} \pi_s z_s$$
$$\text{s.t. } x \in X, \quad u_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b} \quad \forall g, \forall t$$
$$0 \geq (\nu^f)^{\top} \xi_s + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(-\nu^f) \qquad \forall s, \forall f \in \mathcal{F}$$
$$z_s \geq (\nu^o)^{\top} \xi_s + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(c_g - \nu^o) \quad \forall s, \forall o \in \mathcal{O}$$

$$\min_{p} \sum_{g,t} c_{g,t}\, p_{g,t}$$
$$\text{s.t.} \sum_{g} p_g = \xi \qquad [\nu]$$
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$$\min_{p} \sum_{g,[a,b],t} \boldsymbol{\gamma_{g,a,b}} c_{g,t}\, p_{g,a,b,t}$$
$$\text{s.t.} \sum_{g,[a,b]} \boldsymbol{\gamma_{g,a,b}} p_{g,a,b} = \xi \quad [\nu]$$
$$W_g x_{g,a,b} \leq A_g p_{g,a,b} \,\forall g\, \forall [a,b]$$

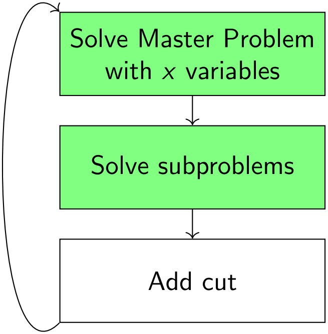
Extended Benders' Decomposition (new)

$$
\begin{aligned}
\min_{x,\gamma} \; & \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s z_s \\
\text{s.t.}\; & x \in X, \quad u_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b} \quad \forall g, \forall t \\
& 0 \geq (\nu^f)^\top \xi_s + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(-\nu^f) \qquad \forall s, \forall f \in \mathcal{F} \\
& z_s \geq (\nu^o)^\top \xi_s + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(c_g - \nu^o) \quad \forall s, \forall o \in \mathcal{O}
\end{aligned}
$$

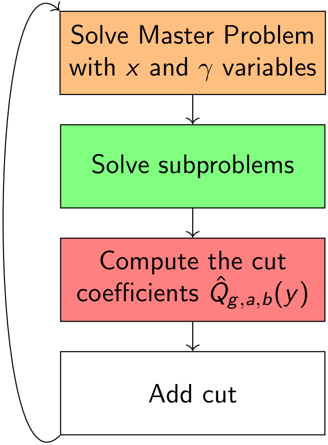
$$
\begin{aligned}
\hat{Q}_{g,a,b}(y) = \min_{p \in \mathbb{R}^T} \quad & y^\top p \\
& W_g x_{g,a,b} \leq A_g p.
\end{aligned}
$$

Algorithm

3bin BD



Extended BD



Heuristic to compute cut coefficients

The new cut coefficients are of the form:

$$\hat{Q}_{g,a,b}(y) = \min_{p \in \mathbb{R}^T} \quad y^\top p$$
$$W_g x_{g,a,b} \leq A_g p$$
$$= \max_{\mu} \quad \mu^\top W_g x_{g,a,b}$$
$$A_g^\top \mu = y.$$

- We developed a heuristic constructing with **deterministic formulas** good dual solutions $\mu_{g,a,b}(y)$ leading to lower bound $\underline{\hat{Q}_{g,a,b}}(y)$ of $\hat{Q}_{g,a,b}(y)$
- The cut with $\underline{\hat{Q}_{g,a,b}}(y)$ as coefficients of $\gamma_{g,a,b}$ is **valid** since it is a lower bound and $\gamma_{g,a,b} \geq 0$.

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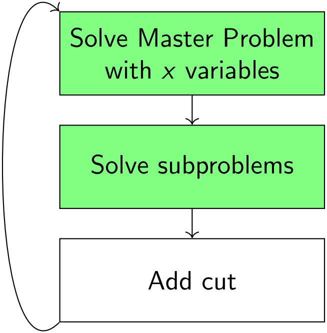
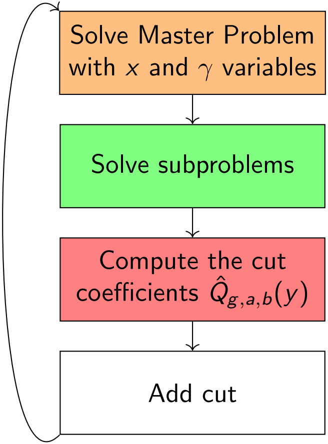
Algorithm

3bin BD

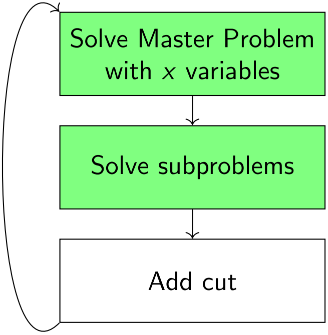


Extended BD

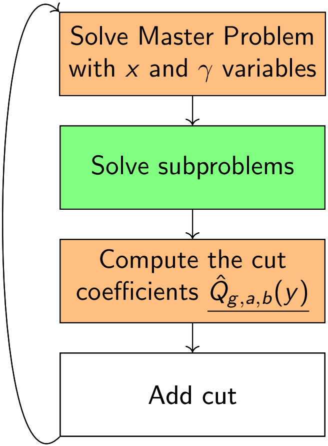


Algorithm

3bin BD



Extended BD



Contents

Instances and Methods

Instance	Generators	Time Steps	Scenarios
SMS++/EDF	10-50-100	24	1-100

Method	First stage variables	Recourse function
`3bin-extensive`	x	primal $Q(x,\xi)$
`3bin BD`	x	dual $Q(x,\xi)$
`Extended BD`	x, γ	$\mathcal{Q}(\gamma,\xi)$

Numerical Results

Units	S	3bin BD	3bin-extensive	Extended BD
10	1	1201s (13/15)	**0s (15/15)**	2s (15/15)
10	25	2290s (1/15)	8s (15/15)	**5s (15/15)**
10	50	3208s (2/15)	22s (15/15)	**7s (15/15)**
10	75	0,84% (0/15)	43s (15/15)	**9s (15/15)**
10	100	1,06% (0/15)	69s (15/15)	**13s (15/15)**
50	1	4,66% (0/15)	**7s (15/15)**	93s (15/15)
50	25	5,52% (0/15)	435s (15/15)	**64s (15/15)**
50	50	4,88% (0/15)	1122s (15/15)	**69s (15/15)**
50	75	5,56% (0/15)	2402s (12/15)	**115s (15/15)**
50	100	5,23% (0/15)	4075s (5/14)	**109s (15/15)**
100	1	8,74% (0/15)	**7s (15/15)**	53s (15/15)
100	25	7,09% (0/15)	1758s (15/15)	**89s (15/15)**
100	50	8,2% (0/15)	4033s (6/15)	**114s (15/15)**
100	75	7,01% (0/15)	92,65% (0/15)	**138s (15/15)**
100	100	7,47% (0/15)	99,24% (0/15)	**220s (15/15)**

Conclusion

- Proposed a novel Benders' Decomposition approach for two-stage Stochastic Unit Commitment.
- Demonstrated superior performance compared to existing methods on large-scale instances.
- The approach is easily extendable to include network constraints.
- It is also adaptable to various stochastic frameworks, such as Robust Optimization, Risk-Averse models (e.g., AVaR), and Distributionally Robust Optimization.

Thank you for your attention!

Azéma, Mathis, Vincent Leclere, and Wim van Ackooij. "Stronger cuts for Benders' decomposition for stochastic Unit Commitment Problems based on interval variables." (preprint)