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LEEDS

Enhanced Benders' Decomposition for Stochastic Unit Commitment Using Interval Variables

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Outline

Section 1. Deterministic Unit Commitment in 3bin formulation

Description

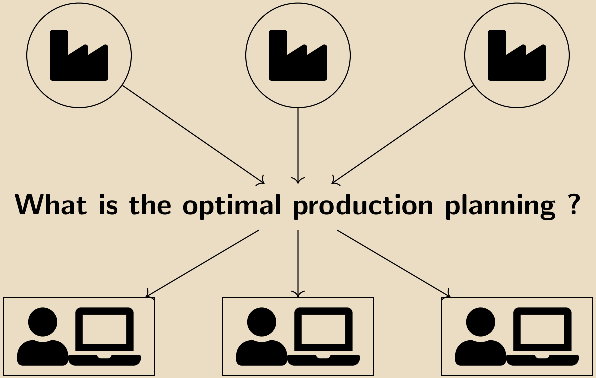

Figure: Unit Commitment

- $\mathcal{G}$: set of generators
- T: time horizon (T=24 for a day)
- $\xi \in \mathbb{R}^T$: vector of demand

Variables

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- $\boldsymbol{p_{g,t}}$: power generation of generator g at time t.

$$\sum_{g \in \mathcal{G}} p_g = \xi$$

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- $\boldsymbol{p_{g,t}}$: power generation of generator g at time t.

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- $\boldsymbol{x_{g,t}} = (\boldsymbol{u_{g,t}}, \boldsymbol{v_{g,t}}, \boldsymbol{w_{g,t}}) \in \{0,1\}^3$
  - $\boldsymbol{u_{g,t}}$: Binary variable equal to 1 if generator g is on at time t.
  - $\boldsymbol{v_{g,t}}$: Binary variable equal to 1 if generator g starts up at time t.
  - $\boldsymbol{w_{g,t}}$: Binary variable equal to 1 if generator g shuts down at time t.

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\boldsymbol{u_{g,t}}$	0	0	1	1	1	0	0	1	1	0	1	1	1	0	0
$\boldsymbol{v_{g,t}}$	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0
$\boldsymbol{w_{g,t}}$	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0

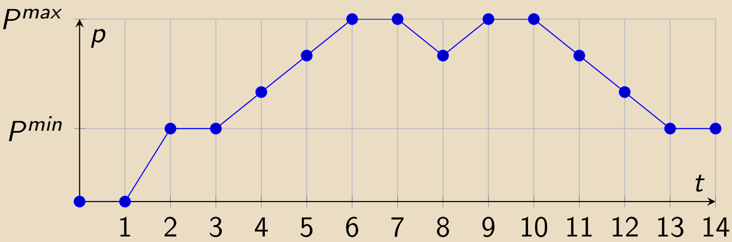
Technical Constraints

Constraints linking the variables x and p

$$W_g x_g \leq A_g p_g$$

Example:

- Capacity constraints: $P_g^{min} u_{g,t} \leq p_{g,t} \leq P_g^{max} u_{g,t}$



u_t	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1

Technical Constraints

Constraints linking the variables x and p

$$W_g x_g \leq A_g p_g \quad \forall g$$

Constraints on the commitment variables $x = (u, v, w)$

$$\begin{aligned} u_{g,t} - u_{g,t-1} &= v_{g,t} - w_{g,t} && \forall g, \forall t \\ x_g &\in X_g && \forall g \end{aligned}$$

Minimum uptime constraint in X_g:

$$\sum_{k \in [t - T_g^U + 1, t]} v_{g,k} \leq u_{g,t}$$

Technical Constraints

Constraints linking the variables x and p

$$W_g x_g \leq A_g p_g \quad \forall g$$

Constraints on the commitment variables $x = (u, v, w)$

$$\begin{aligned} & u_{g,t} - u_{g,t-1} = v_{g,t} - w_{g,t} && \forall g, \forall t \\ & x_g \in X_g && \forall g \end{aligned}$$

Satisfaction of the demand:

$$\sum_{g \in \mathcal{G}} p_g = \xi$$

3-bin formulation

$$
\begin{aligned}
\min_{x,p} \quad & \sum_{g\in\mathcal{G},\, t\in[T]} C_{g,t}^{\top} x_{g,t} + \sum_{g\in\mathcal{G},\, t\in[T]} c_{g,t}\, p_{g,t} \\
\text{s.t.} \quad & \sum_{g\in\mathcal{G}} p_g = \xi \\
& W_g x_g \le A_g p_g && \forall g \\
& u_{g,t} - u_{g,t-1} = v_{g,t} - w_{g,t} && \forall g, \forall t \\
& x_g \in X_g && \forall g \\
& x_{g,t} = (u_{g,t}, v_{g,t}, w_{g,t}) \\
& x_g \in \{0,1\}^{3T},\ p_g \in \mathbb{R}^T.
\end{aligned}
$$

3-bin formulation

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\begin{aligned}
\min_{x,p} \quad & \sum_{g\in\mathcal{G},\, t\in[T]} C_{g,t}^{\top} x_{g,t} + \sum_{g\in\mathcal{G},\, t\in[T]} c_{g,t}\, p_{g,t} \\
\text{s.t.} \quad & \sum_{g\in\mathcal{G}} p_g = \xi \\
& W_g x_g \le A_g p_g && \forall g \\
& x \in X,\ p_g \in \mathbb{R}^T.
\end{aligned}
$$

Section 2. 3-bin approaches for Stochastic Unit Commitment

2-stage Stochastic Unit Commitment

$$
\begin{aligned}
\min_{x,p} \quad & \sum_{g,\,t} C_{g,t}^\top x_{g,t} + \sum_{g,\,t} c_{g,t}\, p_{g,t} \\
\text{s.t.} \quad & \sum_{g} p_g = \boxed{\xi}\,? \\
& W_g x_g \leq A_g p_g && \forall g \\
& x \in X,\; p_g \in \mathbb{R}^T.
\end{aligned}
$$

- Uncertainty on the demand ξ

2-stage Stochastic Unit Commitment

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& W_g x_g \le A_g p_g \qquad\qquad \forall g \\
& x \in X,\ p_g \in \mathbb{R}^T.
\end{aligned}
$$

- Uncertainty on the demand ξ
- **2-stage assumption:**
  - **First-stage:** Commitment decisions $x_{g,t}$ made before uncertainty is revealed
  - **Second-stage:** Generation decisions $p_{g,t}$ made after uncertainty is realized

2-stage Stochastic Unit Commitment

$$\begin{aligned}
\min_{x,p} \quad & \sum_{g,t} C_{g,t}^{\top} x_{g,t} + \sum_{s \in \mathcal{S}} \pi_s \sum_{g,t} c_{g,t}\, p_{s,g,t} \\
\text{s.t.} \quad & \sum_{g} p_{s,g} = \xi_s \quad \forall s \\
& x \in X,\; p_{s,g} \in \mathbb{R}^T.
\end{aligned}$$

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- Set of scenarios ξ_s with probability π_s

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\text{s.t.} \quad & \sum_{g} p_{s,g} = \xi_s \quad \forall s \\
& x \in X,\ p_{s,g} \in \mathbb{R}^T.
\end{aligned}$$

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$\Longrightarrow$ **Extensive formulation: 1 Mixed-Integer Linear Program**

Recourse function: Primal formulation

$\boldsymbol{Q(x,\xi)}$: **recourse value representing the optimal cost of the second stage** (i.e. value of the optimal planning knowing which units are on or off (x) and the demand (ξ))

$$
\begin{aligned}
Q(x,\xi) = \min_{p} \quad & \sum_{g,t} c_{g,t}\, p_{g,t} \\
\text{s.t.} \quad & \sum_{g} p_g = \xi \\
& W_g x_g \leq A_g p_g \quad \forall g
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\text{s.t.} \quad & \sum_{g} p_{s,g} = \xi_s \\
& W_g x_g \leq A_g p_{s,g} \qquad \forall g,\ \forall s \\
& x \in X
\end{aligned}
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& W_g x_g \leq A_g p_g \quad \forall g
\end{aligned}
$$

$$
\begin{aligned}
\min_{x} \quad & \sum_{g,\,t} C_{g,t}^{\top} x_{g,t} + \sum_{s \in \mathcal{S}} \pi_s \min_{\substack{p_s : \sum_g p_{s,g} = \xi_s \\ W_g x_g \leq A_g p_{s,g}}} \sum_{g,\,t} c_{g,t}\, p_{g,t,s} \\
\text{s.t.} \quad & x \in X
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\min_{x} \quad & \sum_{g,t} C_{g,t}^{\top} x_{g,t} + \sum_{s \in \mathcal{S}} \pi_s Q(x, \xi_s) \\
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Primal formulation:

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\text{s.t.} \quad & \sum_{g} p_g = \xi \qquad [\nu] \\
& W_g x_g \leq A_g p_g \, \forall g \, [\mu_g]
\end{aligned}
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Dual formulation:

$$
\begin{aligned}
Q(x,\xi) = \max_{\nu,\mu} \nu^\top \xi + \sum_{g} \mu_g^\top W_g x_g & \\
\text{s.t.} \quad A_g^\top \mu_g + \nu = c_g \quad & \forall g \\
\mu_g \geq 0 \quad & \forall g
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Independent of x and ξ !

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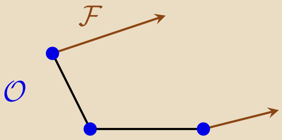

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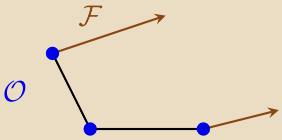

$$
Q(x,\xi) = \begin{cases} +\infty & \text{if } \exists f \in \mathcal{F}, (\nu^f)^\top \xi + \sum_g (\mu_g^f)^\top W_g x_g > 0 \\ \max_{o \in \mathcal{O}} \left((\nu^o)^\top \xi + \sum_g (\mu_g^o)^\top W_g x_g \right) & \text{otherwise.} \end{cases}
$$

3-bin Benders' Decomposition

$$\min_{x} \sum_{g,t} C_{g,t}^{\top} x_{g,t} + \sum_{s} \pi_s Q(x, \xi_s)$$

$$\text{s.t. } x \in X$$

$$Q(x,\xi) =$$

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$$\min_{x} \sum_{g,t} C_{g,t}^{\top} x_{g,t} + \sum_{s} \pi_s z_s$$
$$\text{s.t. } x \in X$$
$$z_s \geq Q(x, \xi_s) \qquad \forall s$$

3-bin Benders' Decomposition

$$\min_{x} \sum_{g,t} C_{g,t}^{\top} x_{g,t} + \sum_{s} \pi_s Q(x, \xi_s)$$
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$$\min_{x} \sum_{g,t} C_{g,t}^{\top} x_{g,t} + \sum_{s} \pi_s z_s$$
$$\text{s.t. } x \in X$$
$$0 \geq (\nu^f)^{\top}\xi_s + \sum_{g} (\mu_g^f)^{\top} W_g x_g \quad \forall s, \forall f \in \mathcal{F}$$

3-bin Benders' Decomposition

$$\min_x \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s Q(x, \xi_s)$$

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3-bin Benders' Decomposition

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$$0 \geq (\nu^f)^\top \xi_s + \sum_g (\mu_g^f)^\top W_g x_g \quad \forall s, \forall f \in \mathcal{F}$$
$$z_s \geq (\nu^o)^\top \xi_s + \sum_g (\mu_g^o)^\top W_g x_g \quad \forall s, \forall o \in \mathcal{O}$$

$\implies$ **Iterative generation of these feasibility and optimality cuts until convergence!**

Section 3. Extended Formulation

Addition of Interval First-Stage Variables

3bin-formulation

$x_{g,t} = (u_{g,t}, v_{g,t}, w_{g,t}) \in \{0,1\}^3$

- $u_{g,t}$: Binary variable equal to 1 if generator g is on at time t.
- $v_{g,t}$: Binary variable equal to 1 if generator g starts up at time t.
- $w_{g,t}$: Binary variable equal to 1 if generator g shuts down at time t.

Extended formulation

$\gamma_{g,a,b}$: equals to 1 if generator g starts up at time a and shuts town at time b

Addition of Interval First-Stage Variables

3bin-formulation

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- $w_{g,t}$: Binary variable equal to 1 if generator g shuts down at time t.

Extended formulation

$\gamma_{g,a,b}$: equals to 1 if generator g starts up at time a and shuts town at time b

$$u_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b}$$

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$u_{g,t}$	0	0	1	1	1	0	0	1	1	0	1	1	1	0	0

$\gamma_{g,2,5} = 1 \quad \gamma_{g,7,9} = 1 \quad \gamma_{g,9,13} = 1$

A new formulation of the recourse function

Proposition

Under some weak assumptions, if x and γ represent the "same" first stage decisions, then:

$$Q(x,\xi) = \mathcal{Q}(\gamma,\xi) := \max_{\nu} \left\{ \nu^\top \xi + \sum_{g,[a,b]} \boldsymbol{\gamma_{g,a,b}} \hat{Q}_{g,a,b}(c_g - \nu) \right\},$$

where:

$$\hat{Q}_{g,a,b}(y) = \min_{p \in \mathbb{R}^T} \quad y^\top p \qquad W_g x_{g,a,b} \leq A_g p.$$

Note: $\hat{Q}_{g,a,b}(y)$ is the optimal cost of the second stage when generator g is on between time a and b and its generation cost is y.

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\boldsymbol{x_{g,2,5,t}}$	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
y	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}

A new formulation of the recourse function

Proposition

Under some weak assumptions, if x and γ represent the "same" first stage decisions, then:

$$Q(x,\xi) = \begin{cases} +\infty & \text{if } \exists f \in \mathcal{F}, (\nu^f)^\top \xi + \sum_g (\mu_g^f)^\top W_g x_g > 0 \\ \max_{o \in \mathcal{O}} \left((\nu^o)^\top \xi + \sum_g (\mu_g^o)^\top W_g x_g \right) & \text{otherwise.} \end{cases}$$

$$= \mathcal{Q}(\gamma,\xi) = \max_{\nu} \nu^\top \xi + \sum_{g,[a,b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(c_g - \nu)$$

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$$= \mathcal{Q}(\gamma,\xi) = \max_{\nu} \nu^\top \xi + \sum_{g,[a,b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(c_g - \nu)$$

$$= \begin{cases} +\infty & \text{if } \exists f \in \mathcal{F}, (\nu^f)^\top \xi + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(-\nu^f) > 0 \\ \max_{o\in\mathcal{O}} \left((\nu^o)^\top \xi + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(c_g - \nu^o)\right) & \text{otherwise.} \end{cases}$$

Extended Benders' Decomposition (new)

$$\min_{x,\gamma} \quad \sum_{g,\,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s Q(x,\xi_s)$$

$$\text{s.t.} \quad x \in X, \quad x = (u,v,w)$$

$$u_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b} \quad \forall g,\ \forall t$$

$$Q(x,\xi) = \mathcal{Q}(\gamma,\xi) =$$

$$\begin{cases} +\infty \quad \text{if } \exists f \in \mathcal{F}, (\nu^f)^\top \xi + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(-\nu^f) > 0 \\ \max_{o\in\mathcal{O}}((\nu^o)^\top \xi + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(c_g - \nu^o)) \quad \text{otherwise.} \end{cases}$$

Extended Benders' Decomposition (new)

$$\min_{x,\gamma} \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s Q(x,\xi_s)$$

$$\text{s.t.} \quad x \in X, \quad x = (u,v,w)$$

$$u_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b} \quad \forall g, \forall t$$

$$Q(x,\xi) = \mathcal{Q}(\gamma,\xi) =$$

$$\begin{cases} +\infty & \text{if } \exists f \in \mathcal{F}, (\nu^f)^\top \xi + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(-\nu^f) > 0 \\ \max_{o\in\mathcal{O}} \left((\nu^o)^\top \xi + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(c_g - \nu^o)\right) & \text{otherwise.} \end{cases}$$

$$\min_{x,\gamma} \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s z_s$$

$$\text{s.t.}\ x \in X, \quad u_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b} \quad \forall g, \forall t$$

$$z_s \geq Q(x,\xi_s) \quad \forall s$$

Extended Benders' Decomposition (new)

$$\min_{x,\gamma} \quad \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s Q(x, \xi_s)$$
$$\text{s.t.} \quad x \in X, \quad x = (u, v, w)$$
$$u_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b} \quad \forall g, \forall t$$

$$Q(x,\xi) = \mathcal{Q}(\gamma,\xi) = $$
$$\begin{cases} +\infty & \text{if } \exists f \in \mathcal{F}, (\nu^f)^\top \xi + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(-\nu^f) > 0 \\ \max_{o\in\mathcal{O}}((\nu^o)^\top \xi + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(c_g - \nu^o)) & \text{otherwise.} \end{cases}$$

$$\min_{x,\gamma} \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s z_s$$
$$\text{s.t.}\, x \in X, \quad u_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b} \quad \forall g, \forall t$$
$$0 \geq (\nu^f)^\top \xi_s + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(-\nu^f) \qquad \forall s, \forall f \in \mathcal{F}$$

Extended Benders' Decomposition (new)

$$\min_{x,\gamma} \quad \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s Q(x, \xi_s)$$

$$\text{s.t.} \quad x \in X, \quad x = (u, v, w)$$

$$u_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b} \quad \forall g, \forall t$$

$$Q(x,\xi) = \mathcal{Q}(\gamma,\xi) =$$

$$\begin{cases} +\infty & \text{if } \exists f \in \mathcal{F}, (\nu^f)^\top \xi + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(-\nu^f) > 0 \\ \max_{o\in\mathcal{O}}((\nu^o)^\top \xi + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(c_g - \nu^o)) & \text{otherwise.} \end{cases}$$

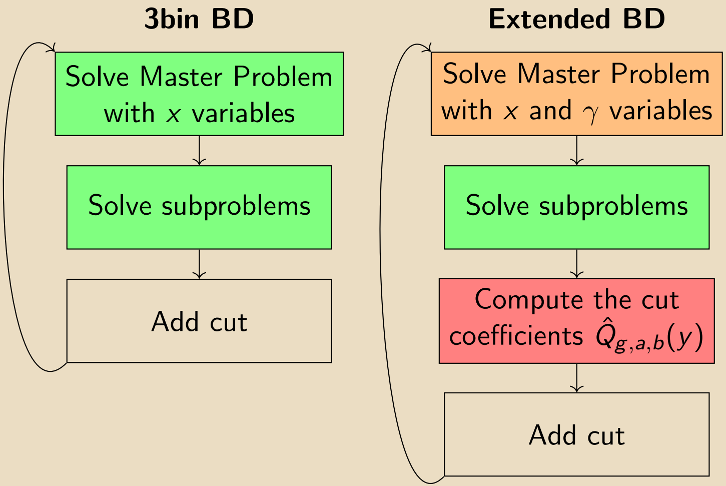
$$\min_{x,\gamma} \sum_{g,t} C_{g,t}^\top x_{g,t} + \sum_s \pi_s z_s$$

$$\text{s.t.}\; x \in X, \quad u_{g,t} = \sum_{[a,b]\ni t} \gamma_{g,a,b} \quad \forall g, \forall t$$

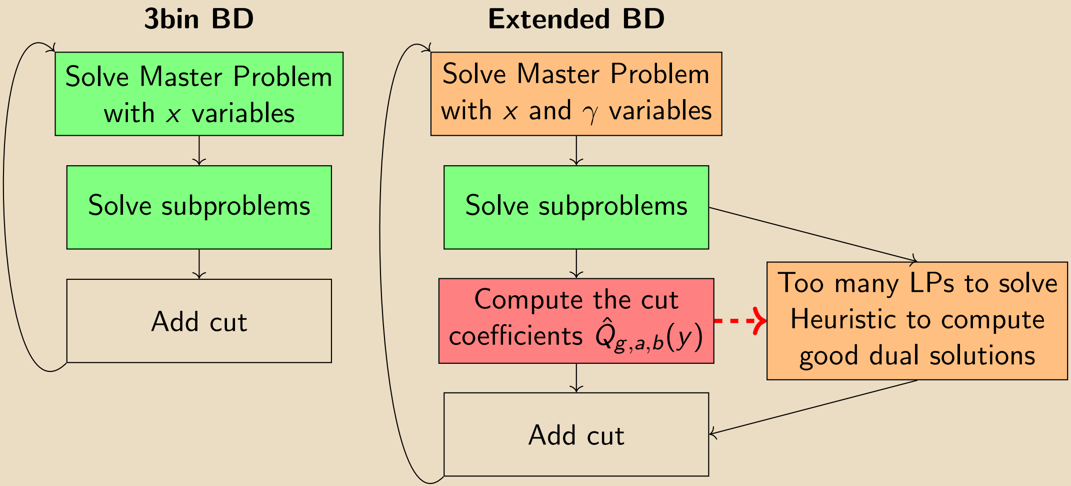
$$0 \geq (\nu^f)^\top \xi_s + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(-\nu^f) \qquad \forall s, \forall f \in \mathcal{F}$$

$$z_s \geq (\nu^o)^\top \xi_s + \sum_{g,[a:b]} \gamma_{g,a,b} \hat{Q}_{g,a,b}(c_g - \nu^o) \quad \forall s, \forall o \in \mathcal{O}$$

Summary


3bin BD
Solve Master Problem with x variables
Solve subproblems
Add cut
Extended BD
Solve Master Problem with x and γ variables
Solve subproblems
Compute the cut coefficients $\hat{Q}_{g,a,b}(y)$
Add cut

Summary


3bin BD
Solve Master Problem with x variables
Solve subproblems
Add cut
Extended BD
Solve Master Problem with x and γ variables
Solve subproblems
Compute the cut coefficients $\hat{Q}_{g,a,b}(y)$
Too many LPs to solve
Heuristic to compute good dual solutions
Add cut

Section 4. Computational Experiments

Instances and Methods

Instance	Generators	Time Steps	Scenarios
SMS++/EDF	10-20-50	24	1-100

Method	First stage variables	Recourse function
`3bin-extensive`	x	primal $Q(x, \xi)$
`3bin BD`	x	dual $Q(x, \xi)$
`Extended BD`	x, γ	$\mathcal{Q}(\gamma, \xi)$

Numerical Results

Units	S	3bin BD	3bin-extensive	Extended BD
10	1	$\times$	**0s**	3s
10	25	$\times$	8s	**7s**
10	50	$\times$	20s	**8s**
10	75	$\times$	37s	**11s**
10	100	$\times$	59s	**14s**
20	1	$\times$	**2s**	33s
20	25	$\times$	41s	**40s**
20	50	$\times$	104s	**36s**
20	75	$\times$	367s	**48s**
20	100	$\times$	542s	**61s**
50	1	$\times$	**6s**	66s
50	25	$\times$	333s	**54s**
50	50	$\times$	816s	**60s**
50	75	$\times$	2167s	**84s**
50	100	$\times$	2627s	**85s**

Section 5. Conclusion

Conclusion

- Proposed a novel Benders' Decomposition approach for two-stage Stochastic Unit Commitment.
- Demonstrated superior performance compared to existing methods on large-scale instances.
- The approach is easily extendable to include network constraints.
- It is also adaptable to various stochastic frameworks, such as Robust Optimization, Risk-Averse models (e.g., AVaR), and Distributionally Robust Optimization.

Thank You!